Supplemental Materials to Lab 4 of QM2

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1 Exact Change and Differentials

The *exact change* in x and y is Δx and Δy respectively, and *differential* dx and dy are related through $dy = f'(x)dx$. In particular, note that

$$dx = \Delta x \tag{1}$$

$$dy \approx \Delta y \tag{2}$$

2 Relative Change and Percentage Change

- The *relative change* is $\frac{\Delta x}{x}$
- The *percentage change* is $\frac{\Delta x}{x} \times 100$

3 Example: Log-Level Model

Consider the following model:

$$\log(y) = b_0 + b_1 x \tag{3}$$

- By definition of logarithm: $y = e^{b_0 + b_1 x}$;
- By basic derivation: $\frac{dy}{dx} = b_1 e^{b_0 + b_1 x}$;
- Recognize that $e^{b_0+b_1x}$ is just $y \Leftrightarrow \frac{dy}{dx} = b_1 e^{b_0+b_1x} = b_1 y$
- Basic algebra to make LHS a percentage change: $\frac{dy}{y}100 = 100 b_1 dx$;
- Interpretation: A *one unit change* in x leads to $100b_1$ *percentage change* in y

4 Example: Quadratic Model

Consider the following model:

$$y = b_0 + b_1 \ln(x) + b_2 \ln(x)^2 \tag{4}$$

- By basic derivation: $dy/dx = b_1 \frac{1}{x} + 2b_2 \frac{\ln(x)}{x}$
- Basic algebra: $dy = [b_1 + 2b_2 \ln(x)] \frac{dx}{x}$
- A second round of basic algebra: $dy = \frac{b_1 + 2b_2 \ln(x)}{100} \frac{dx}{x} 100$
- Interpretation: A *one percentage change* in x leads to $\frac{b_1 + 2b_2 \ln(x)}{100}$ *unit change* in y

5 Example: Turning Point

Consider the following estimating equation:

$$wage = 3.73 + 0.298 exp - 0.0061 exp^2 \tag{5}$$

- Take derivative:

$$\frac{d(wage)}{d(exp)} = 0.298 - (2)(0.0061) exp \tag{6}$$

- By first order condition, set $\frac{d(wage)}{d exp} \equiv 0$, which yields: $0.298 - (2)(0.0061) exp = 0$
- Solving the equation above gives us

$$exp = \frac{0.298}{(2)(0.0061)} = 24.4 \tag{7}$$

- Interpretation: The turning point is when experience takes a value of 24.4.