Supplemental Materials to Lab 8 of QM2

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1 Multilevel Linear Models

Consider the following model with varying intercept and varying slope:

$$y_i \sim \mathcal{N}\left(\alpha_{j[i]} + \beta_{j[i]} x_i, \sigma_y^2\right), \; i = 1, ..., n \tag{1}$$

$$\begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\alpha \\ \mu_\beta \end{pmatrix}, \begin{pmatrix} \sigma_\alpha^2 & \rho\sigma_\alpha\sigma_\beta \\ \rho\sigma_\alpha\sigma_\beta & \sigma_\beta^2 \end{pmatrix}\right), \; j = 1, ..., J \tag{2}$$

where i indicates countries and j indicates groups.

2 Model Varying Slopes as Interactions

A multilevel model with varying slope can be written as a model including multiple interactions. Suppose you have the following model:

$$\begin{aligned} y = \beta_0 + \beta_1 SD + \beta_2 NY + \beta_3 age + \beta_4 \left(age \times SD\right) + \beta_5 \left(age \times NY\right) \\ + \beta_6 age^2 + \beta_7 \left(age^2 \times SD\right) + \beta_8 \left(age^2 \times NY\right) \end{aligned} \tag{3}$$

With this model, we allow `age` to have a quadratic relationship with y, and we allow it to vary in slope, center, and intercept by cities. One way to think about Eq (3) is that there are multiple separate regression lines.

For the baseline (reference) category,

$$\hat{y} = \beta_0 + \beta_3 age + \beta_6 age^2 \tag{4}$$

For SD,

$$\hat{y} = \beta_0 + \beta_1 + \beta_3 age + \beta_4 age + \beta_6 age^2 + \beta 7 age^2 \tag{5}$$

$$= (\beta_0 + \beta_1) + (\beta_3 + \beta_4) age + (\beta_6 + \beta 7) age^2 \tag{6}$$

For NY,

$$\hat{y} = \beta_0 + \beta_2 + \beta_3 age + \beta_5 age + \beta_6 age^2 + \beta 8 age^2 \tag{7}$$

$$= (\beta_0 + \beta_2) + (\beta_3 + \beta_5) age + (\beta_6 + \beta 8) age^2 \tag{8}$$

That is,

- β_0 is the intercept of the baseline category when age is set to zero
- For SD, the intercept is $\beta_0 + \beta_1$, and the quadratic relationship is captured by $\beta_6 + \beta_7$, and the center shift is captured by $\beta_3 + \beta_4$
- For NY, the intercept is $\beta_0 + \beta_2$, and the quadratic relationship is captured by $\beta_6 + \beta_8$, and the center shift is captured by $\beta_3 + \beta_5$
- For SD and NY, the coefficients are *relative change* compared to the baseline category

3 Interpretation of Interaction Terms

There are two things that you need to bear in mind while interpreting the results of an interaction term:

- Is there an interaction (that is, are the slopes *parallel* or not)? We can know the answer by checking whether the coefficient on the interaction term is significant or not
- What is the nature of the interaction? To see this, we need to do a little bit more math and coding

Consider the following classical pedagogical model, in which wage is a function of age and gender:

$$wage = \beta_0 + \beta_1 age + \beta_2 male + \beta_3(age \times male) + e \tag{9}$$

The conditional effect (`Stata` users insist calling it "marginal effect") of age on wage is

$$\frac{\partial(\widehat{wage})}{\partial(age)} = \beta_1 + \beta_3(male) \tag{10}$$

- To test whether the effect of age on wage for males is statistically different from zero, we test $H_0^{(a)} : \beta_1 + \beta_3 = 0$;
- To test whether the effect of age on wage for females is statistically different from zero, we test $H_0^{(b)} : \beta_1 = 0$;
- To test whether the effect of age on wage differs in terms of gender, we test $H_0^{(c)} : \beta_3 = 0$.

4 Cutting-Edge Work on Interaction Models

- Esarey and Sumner (2017). Marginal Effects in Interaction Models: Determining and Controlling the False Positive Rate. *CPS*, forthcoming.
- Hainmueller, Mummolo and Xu (2016). How Much Should We Trust Estimates from Multiplicative Interaction Models? *Working paper.*

5 Appendix: Code

Posted on the course website.