GPCO 453: Quantitative Methods I

Sec 07: Samples

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Contact Information

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The teaching staff is a team!

Professor Garg	Tu	1300-1500 (RBC 1303)
Shane Xuan	M	1100-1200 (SSB 332)
	M	1530-1630 (SSB 332)
Joanna Valle-luna	Tu	1700-1800 (RBC 3131)
	Th	1300-1400 (RBC 3131)
Daniel Rust	F	1100-1230 (RBC 3213)

Roadmap

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- Sample proportion

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- The distribution is hence $\hat{p} \sim \mathcal{N}(0.76, 0.0214^2)$

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- Commonly used confidence level includes 68% ($z = 1$), 90% ($z = 1.645$), and 95% ($z = 1.96$).

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- The solution is

$$\overline{x} \pm z * \frac{\sigma}{\sqrt{n}} \tag{5}$$

$$= 127 \pm 1.96 * \frac{19}{\sqrt{1000}} \tag{6}$$

$$= (125.82, 128.18)$$

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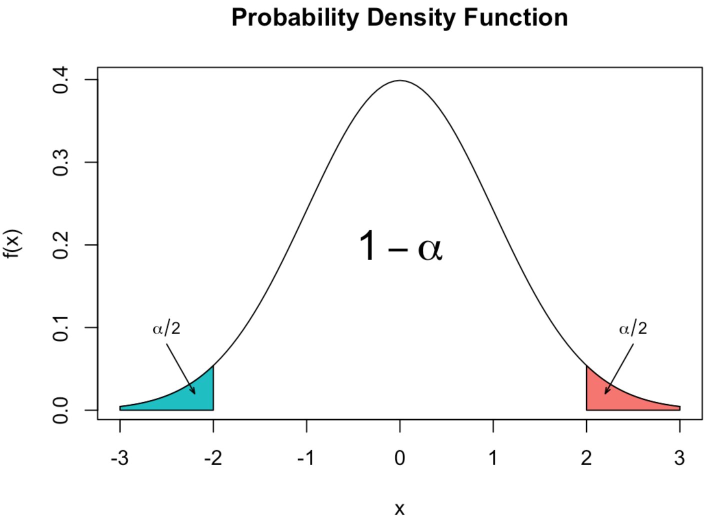
- That is, in the case in which σ is unknown, we can bootstrap σ with s, as long as we use t instead of z

t-distribution

- (Rule of thumb) We use z-table when $n \geq 30$, and t-table when $n < 30$. The t-statistics is calculated by

$$t_{\overline{x}} = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

t-distribution



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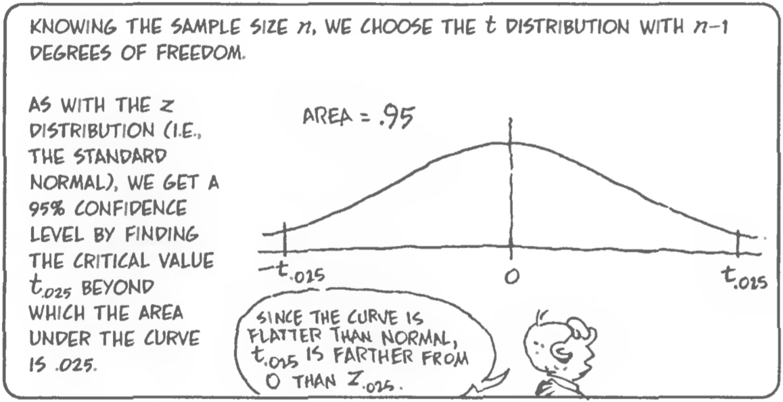

The cartoon guide to statistics (Larry Gonick)

t-distribution

FOR A $(1-\alpha)\cdot 100\%$ CONFIDENCE INTERVAL, WE FIND THE CRITICAL VALUE $t_{\frac{\alpha}{2}}$ SUCH THAT $Pr(t \geq t_{\frac{\alpha}{2}}) = \frac{\alpha}{2}$. HERE IS A SHORT TABLE OF CRITICAL VALUES FOR THE t DISTRIBUTION:

	$1-\alpha$	.80	.90	.95	.99
	α	.20	.10	.05	.01
	$\alpha/2$	.10	.05	.025	.005
DEGREES OF FREEDOM	1	3.09	6.31	12.71	63.66
	10	1.37	1.81	2.23	4.14
	30	1.31	1.70	2.04	2.75
	100	1.29	1.66	1.98	2.63
	∞	1.28	1.65	1.96	2.58

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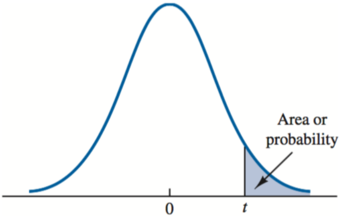

Degrees of Freedom	Area in Upper Tail .20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250

t-distribution

Table: When t-table only gives you information about Area in Upper Tail

$\frac{\alpha}{2}$	α	$1-\alpha$
0.05	0.1	90%
0.025	0.05	95%
0.01	0.02	98%
0.005	0.01	99%