GPCO 453: Quantitative Methods I

Sec 10: Hypothesis Testing, III

Shane Xinyang Xuan[1]
ShaneXuan.com

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[1]Department of Political Science, UC San Diego, 9500 Gilman Drive #0521.

Contact Information

Shane Xinyang Xuan

xxuan@ucsd.edu

My office hours for the rest of the quarter

12/4	M	1100-1230 (SSB 332)
12/4	M	1330-1400 (SSB 332)
12/12	Tu	1100-1130 (SSB 332)

Interval Estimate of a Population Variance

- χ^2 are based on $n-1$ d.f. and $(1-\alpha)$ confidence level

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}} \tag{1}$$

Test Statistic for Hyp. Tests about a Population Variance

- χ^2 follows a chi-square distribution with $n-1$ d.f.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \tag{2}$$

Hypothesis Tests about a Population Variance, One-tailed

- A company produces metal pipes and claims that the standard deviation of the length is at most 1.2 cm. One of its clients decides to test this claim by taking a sample of 25 pipes. They found that the standard deviation of the sample is 1.5 cm.

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- Calculate the test statistics

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$
$$= \frac{(25-1)(1.5)^2}{(1.2)^2} = 37.5$$

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- Note that $\chi^2 > \chi^2_\alpha \rightarrow$ We reject the null

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$$\begin{aligned} \chi^2 &= \frac{(n-1)s^2}{\sigma^2} \\ &= \frac{(25-1)(17.7)^2}{225} = 33.42 \end{aligned}$$

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- Note that $\chi^2_{0.975} < \chi^2 < \chi^2_{0.025} \rightarrow$ We fail to reject the null

Hypothesis Tests about Two Population Variances

- Denoting the population with the larger sample variance as population 1, the test statistic has an F-distribution with $n_1 - 1$ degrees of freedom for the numerator and $n_2 - 1$ degrees of freedom for the denominator.

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- Denoting the population with the larger sample variance as population 1, the test statistic has an F-distribution with $n_1 - 1$ degrees of freedom for the numerator and $n_2 - 1$ degrees of freedom for the denominator.
- The test statistic (when $\sigma_1^2 = \sigma_2^2$) is

$$F = \frac{s_1^2}{s_2^2} \tag{3}$$

Example: Two Population Variances

- Suppose $n_1 = 31$, and $n_2 = 26$, and

$$F = \frac{s_1^2}{s_2^2} = \frac{120}{80} = 1.5 \tag{4}$$

We want to test at 95% confidence level:

$$H_0 : \sigma_1^2 \leq \sigma_2^2$$
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- We find that F distribution with 30 numerator degrees of freedom and 25 denominator degrees of freedom has $F_{.05} = 1.92$ F distribution
- Since the test statistic F is less than the critical value $F_{.05}$, we conclude that we fail to reject H_0.

Appendix: χ^2−table

back

Degrees of Freedom	Area in Upper Tail .99	.975	.95	.90	.10	.05	.025	.01
1	.000	.001	.004	.016	2.706	3.841	5.024	6.635
2	.020	.051	.103	.211	4.605	5.991	7.378	9.210
3	.115	.216	.352	.584	6.251	7.815	9.348	11.345
4	.297	.484	.711	1.064	7.779	9.488	11.143	13.277
5	.554	.831	1.145	1.610	9.236	11.070	12.832	15.086
6	.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.647	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.041	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980

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Appendix: F distribution

◂ back

Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom 10	15	20	25	30
10	.10	2.32	2.24	2.20	2.17	2.16
	.05	2.98	2.85	2.77	2.73	2.70
	.025	3.72	3.52	3.42	3.35	3.31
	.01	4.85	4.56	4.41	4.31	4.25
15	.10	2.06	1.97	1.92	1.89	1.87
	.05	2.54	2.40	2.33	2.28	2.25
	.025	3.06	2.86	2.76	2.69	2.64
	.01	3.80	3.52	3.37	3.28	3.21
20	.10	1.94	1.84	1.79	1.76	1.74
	.05	2.35	2.20	2.12	2.07	2.04
	.025	2.77	2.57	2.46	2.40	2.35
	.01	3.37	3.09	2.94	2.84	2.78
25	.10	1.87	1.77	1.72	1.68	1.66
	.05	2.24	2.09	2.01	1.96	1.92
	.025	2.61	2.41	2.30	2.23	2.18
	.01	3.13	2.85	2.70	2.60	2.54
30	.10	1.82	1.72	1.67	1.63	1.61
	.05	2.16	2.01	1.93	1.88	1.84
	.025	2.51	2.31	2.20	2.12	2.07
	.01	2.98	2.70	2.55	2.45	2.39