GPCO 453: Quantitative Methods I

Sec 08: Hypothesis Testing

Shane Xinyang Xuan[1]
ShaneXuan.com

November 14, 2017

[1]Department of Political Science, UC San Diego, 9500 Gilman Drive #0521.

Contact Information

Shane Xinyang Xuan

xxuan@ucsd.edu

The teaching staff is a team!

Professor Garg	Tu	1300-1500 (RBC 1303)
Shane Xuan	M	1100-1200 (SSB 332)
	M	1530-1630 (SSB 332)
Joanna Valle-luna	Tu	1700-1800 (RBC 3131)
	Th	1300-1400 (RBC 3131)
Daniel Rust	F	1100-1230 (RBC 3213)

Roadmap

In this section, we cover the basics for hypothesis testing:

- Step 1: State H_0 and H_A

Roadmap

In this section, we cover the basics for hypothesis testing:

- Step 1: State H_0 and H_A
- Step 2: State $1 - \alpha$ (level of significance)

Roadmap

In this section, we cover the basics for hypothesis testing:

- Step 1: State H_0 and H_A
- Step 2: State $1 - \alpha$ (level of significance)
- Step 3: Compute z score (or t score)

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \qquad \text{mean} \qquad (1)$$

$$= \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \qquad \text{proportion} \qquad (2)$$

Roadmap

In this section, we cover the basics for hypothesis testing:

- Step 1: State H_0 and H_A
- Step 2: State $1 - \alpha$ (level of significance)
- Step 3: Compute z score (or t score)

$$z = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}} \qquad \text{mean} \qquad (1)$$

$$= \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \qquad \text{proportion} \qquad (2)$$

- Step 4: Compute p value (or find the rejection area)

Roadmap

In this section, we cover the basics for hypothesis testing:

- Step 1: State H_0 and H_A
- Step 2: State $1-\alpha$ (level of significance)
- Step 3: Compute z score (or t score)

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \qquad \text{mean} \qquad (1)$$

$$= \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \qquad \text{proportion} \qquad (2)$$

- Step 4: Compute p value (or find the rejection area)
- Step 5: Perform the test

Roadmap

In this section, we cover the basics for hypothesis testing:

- Step 1: State H_0 and H_A
- Step 2: State $1-\alpha$ (level of significance)
- Step 3: Compute z score (or t score)

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \quad \text{mean} \tag{1}$$

$$= \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \quad \text{proportion} \tag{2}$$

- Step 4: Compute p value (or find the rejection area)
- Step 5: Perform the test
- Step 6: Conclusion: Either reject the null, or fail to reject null

Example: Hypothesis Test for a Mean with Known σ

- The null hypothesis is that the mean weight for men is 191 pounds in 2006. The alternative hypothesis is that the mean weight for men in 2006 is more than 191 pounds.

Example: Hypothesis Test for a Mean with Known σ

- The null hypothesis is that the mean weight for men is 191 pounds in 2006. The alternative hypothesis is that the mean weight for men in 2006 is more than 191 pounds.
- Suppose you have the following information: $n = 100$, $\overline{x} = 197.1$, and $\sigma = 25.6$.

Example: Hypothesis Test for a Mean with Known σ

- The null hypothesis is that the mean weight for men is 191 pounds in 2006. The alternative hypothesis is that the mean weight for men in 2006 is more than 191 pounds.
- Suppose you have the following information: $n = 100$, $\overline{x} = 197.1$, and $\sigma = 25.6$.
- (Step 1) $H_0 : \mu = 191$, $H_A : \mu > 191$

Example: Hypothesis Test for a Mean with Known σ

- The null hypothesis is that the mean weight for men is 191 pounds in 2006. The alternative hypothesis is that the mean weight for men in 2006 is more than 191 pounds.
- Suppose you have the following information: $n = 100$, $\overline{x} = 197.1$, and $\sigma = 25.6$.
- (Step 1) $H_0 : \mu = 191$, $H_A : \mu > 191$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)

Example: Hypothesis Test for a Mean with Known σ

- The null hypothesis is that the mean weight for men is 191 pounds in 2006. The alternative hypothesis is that the mean weight for men in 2006 is more than 191 pounds.
- Suppose you have the following information: $n = 100$, $\overline{x} = 197.1$, and $\sigma = 25.6$.
- (Step 1) $H_0 : \mu = 191$, $H_A : \mu > 191$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute $z = \frac{197.1 - 191}{25.6/\sqrt{100}} = 2.38$

Example: Hypothesis Test for a Mean with Known σ

- The null hypothesis is that the mean weight for men is 191 pounds in 2006. The alternative hypothesis is that the mean weight for men in 2006 is more than 191 pounds.
- Suppose you have the following information: $n = 100$, $\overline{x} = 197.1$, and $\sigma = 25.6$.
- (Step 1) $H_0 : \mu = 191$, $H_A : \mu > 191$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute $z = \frac{197.1-191}{25.6/\sqrt{100}} = 2.38$
- (Step 4) Compute $\Pr(X > 197.1) = \Pr(z > 2.38) = 0.0087$

Example: Hypothesis Test for a Mean with Known σ

- The null hypothesis is that the mean weight for men is 191 pounds in 2006. The alternative hypothesis is that the mean weight for men in 2006 is more than 191 pounds.
- Suppose you have the following information: $n = 100$, $\overline{x} = 197.1$, and $\sigma = 25.6$.
- (Step 1) $H_0 : \mu = 191$, $H_A : \mu > 191$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute $z = \frac{197.1-191}{25.6/\sqrt{100}} = 2.38$
- (Step 4) Compute $\Pr(X > 197.1) = \Pr(z > 2.38) = 0.0087$
- (Step 5) Compare p-value to $\alpha = 0.05$, we find that given the null hypothesis, the sample findings are unlikely

Example: Hypothesis Test for a Mean with Known σ

- The null hypothesis is that the mean weight for men is 191 pounds in 2006. The alternative hypothesis is that the mean weight for men in 2006 is more than 191 pounds.
- Suppose you have the following information: $n = 100$, $\overline{x} = 197.1$, and $\sigma = 25.6$.
- (Step 1) $H_0 : \mu = 191$, $H_A : \mu > 191$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute $z = \frac{197.1 - 191}{25.6/\sqrt{100}} = 2.38$
- (Step 4) Compute $\Pr(X > 197.1) = \Pr(z > 2.38) = 0.0087$
- (Step 5) Compare p-value to $\alpha = 0.05$, we find that given the null hypothesis, the sample findings are unlikely
- (Step 6) We conclude that we can reject the null hypothesis

What If It's a Tw-Tailed Test?

- In Step 4, simply double the result and treat it as your p-value

Example: Hypothesis Test for a Proportion

- Suppose that $n = 100$, $p = .8$, and $\hat{p} = .73$

Example: Hypothesis Test for a Proportion

- Suppose that $n = 100$, $p = .8$, and $\hat{p} = .73$
- (Step 1) $H_0 : p = .8$, $H_A : p \neq .8$

Example: Hypothesis Test for a Proportion

- Suppose that $n = 100$, $p = .8$, and $\hat{p} = .73$
- (Step 1) $H_0 : p = .8$, $H_A : p \neq .8$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)

Example: Hypothesis Test for a Proportion

- Suppose that $n = 100$, $p = .8$, and $\hat{p} = .73$
- (Step 1) $H_0 : p = .8$, $H_A : p \neq .8$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute z-score

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} = \frac{.73 - .8}{\sqrt{(.8)(.2)/100}} = -1.75 \qquad (3)$$

Example: Hypothesis Test for a Proportion

- Suppose that $n = 100$, $p = .8$, and $\hat{p} = .73$
- (Step 1) $H_0 : p = .8$, $H_A : p \neq .8$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute z-score

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} = \frac{.73 - .8}{\sqrt{(.8)(.2)/100}} = -1.75 \qquad (3)$$

- (Step 4) Compute p-value

$$\Pr(z < -1.75) + \Pr(z > 1.75) = 0.08 \qquad (4)$$

Example: Hypothesis Test for a Proportion

- Suppose that $n = 100$, $p = .8$, and $\hat{p} = .73$
- (Step 1) $H_0 : p = .8$, $H_A : p \neq .8$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute z-score

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} = \frac{.73 - .8}{\sqrt{(.8)(.2)/100}} = -1.75 \qquad (3)$$

- (Step 4) Compute p-value

$$\Pr(z < -1.75) + \Pr(z > 1.75) = 0.08 \qquad (4)$$

- (Step 5) We find that p-value is larger than $\alpha = 0.05$

Example: Hypothesis Test for a Proportion

- Suppose that $n = 100$, $p = .8$, and $\hat{p} = .73$
- (Step 1) $H_0 : p = .8$, $H_A : p \neq .8$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute z-score

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}} = \frac{.73 - .8}{\sqrt{(.8)(.2)/100}} = -1.75 \tag{3}$$

- (Step 4) Compute p-value

$$\Pr(z < -1.75) + \Pr(z > 1.75) = 0.08 \tag{4}$$

- (Step 5) We find that p-value is larger than $\alpha = 0.05$
- (Step 6) We conclude that we fail to reject the null hypothesis

What if n is Small and σ is Unknown?

- Use a t-test!

What if n is Small and σ is Unknown?

- Use a t-test!
- Remember from last time that $t = \frac{\overline{x}-\mu}{s/\sqrt{n}}$

What if n is Small and σ is Unknown?

- Use a t-test!
- Remember from last time that $t = \frac{\overline{x} - \mu}{s/\sqrt{n}}$
- Suppose that $n = 100, \overline{x} = 315.4$, and $s = 43.2$

What if n is Small and σ is Unknown?

- Use a t-test!
- Remember from last time that $t = \frac{\overline{x}-\mu}{s/\sqrt{n}}$
- Suppose that $n = 100, \overline{x} = 315.4$, and $s = 43.2$
- (Step 1) $H_0 : \mu = 300$, $H_A : \mu \neq 300$

What if n is Small and σ is Unknown?

- Use a t-test!
- Remember from last time that $t = \frac{\overline{x}-\mu}{s/\sqrt{n}}$
- Suppose that $n = 100, \overline{x} = 315.4$, and $s = 43.2$
- (Step 1) $H_0 : \mu = 300$, $H_A : \mu \neq 300$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)

What if n is Small and σ is Unknown?

- Use a t-test!
- Remember from last time that $t = \frac{\overline{x}-\mu}{s/\sqrt{n}}$
- Suppose that $n = 100, \overline{x} = 315.4$, and $s = 43.2$
- (Step 1) $H_0 : \mu = 300$, $H_A : \mu \neq 300$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute t-statistic

$$t = \frac{315.4 - 300}{43.2/\sqrt{100}} = 3.5648 \tag{5}$$

What if n is Small and σ is Unknown?

- Use a t-test!
- Remember from last time that $t = \frac{\overline{x}-\mu}{s/\sqrt{n}}$
- Suppose that $n = 100, \overline{x} = 315.4$, and $s = 43.2$
- (Step 1) $H_0 : \mu = 300$, $H_A : \mu \neq 300$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute t-statistic

$$t = \frac{315.4 - 300}{43.2/\sqrt{100}} = 3.5648 \tag{5}$$

- (Step 4) Find the rejection region $\rightarrow t_{\alpha/2}^{n-1} = 1.984$ t-table

What if n is Small and σ is Unknown?

- Use a t-test!
- Remember from last time that $t = \frac{\overline{x}-\mu}{s/\sqrt{n}}$
- Suppose that $n = 100, \overline{x} = 315.4$, and $s = 43.2$
- (Step 1) $H_0 : \mu = 300$, $H_A : \mu \neq 300$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute t-statistic

$$t = \frac{315.4 - 300}{43.2/\sqrt{100}} = 3.5648 \quad (5)$$

- (Step 4) Find the rejection region $\rightarrow t^{n-1}_{\alpha/2} = 1.984$ t-table
- (Step 5) We find that the test statistic (t) is larger than $t^{n-1}_{\alpha/2}$

What if n is Small and σ is Unknown?

- Use a t-test!
- Remember from last time that $t = \frac{\overline{x}-\mu}{s/\sqrt{n}}$
- Suppose that $n = 100, \overline{x} = 315.4$, and $s = 43.2$
- (Step 1) $H_0 : \mu = 300$, $H_A : \mu \neq 300$
- (Step 2) We pick 95% confidence level ($\alpha = 0.05$)
- (Step 3) Compute t-statistic

$$t = \frac{315.4 - 300}{43.2/\sqrt{100}} = 3.5648 \tag{5}$$

- (Step 4) Find the rejection region $\rightarrow t_{\alpha/2}^{n-1} = 1.984$ t-table
- (Step 5) We find that the test statistic (t) is larger than $t_{\alpha/2}^{n-1}$
- (Step 6) We conclude that we can reject the null hypothesis

One-tailed and Two-tailed t-tests

Table: For one-tailed tests, look at α; for two-tailed tests, look at $\alpha/2$

t-table

Test	$1-\alpha$	α	$\alpha/2$	Area in the Upper Tail
One-tailed, 95% CI	95%	**5%**	2.5%	0.050
Two-tailed, 95% CI	95%	5%	**2.5%**	0.025
One-tailed, 99% CI	99%	**1%**	.5%	0.010
Two-tailed, 99% CI	99%	1%	**.5%**	0.005

Conclusion

- Are we dealing with mean or proportion?

Conclusion

- Are we dealing with mean or proportion?
- Should we use z-score or t-score?

Conclusion

- Are we dealing with mean or proportion?
- Should we use z-score or t-score?
- Should the test be one-tailed or two-tailed?

Appendix: $t-$table

◂ back

Degrees of Freedom	Area in Upper Tail .20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
⋮	⋮	⋮	⋮	⋮	⋮	⋮
60	.848	1.296	1.671	2.000	2.390	2.660
61	.848	1.296	1.670	2.000	2.389	2.659
62	.847	1.295	1.670	1.999	2.388	2.657
63	.847	1.295	1.669	1.998	2.387	2.656
64	.847	1.295	1.669	1.998	2.386	2.655
65	.847	1.295	1.669	1.997	2.385	2.654
66	.847	1.295	1.668	1.997	2.384	2.652
67	.847	1.294	1.668	1.996	2.383	2.651
68	.847	1.294	1.668	1.995	2.382	2.650
69	.847	1.294	1.667	1.995	2.382	2.649
⋮	⋮	⋮	⋮	⋮	⋮	⋮
90	.846	1.291	1.662	1.987	2.368	2.632
91	.846	1.291	1.662	1.986	2.368	2.631
92	.846	1.291	1.662	1.986	2.368	2.630
93	.846	1.291	1.661	1.986	2.367	2.630
94	.845	1.291	1.661	1.986	2.367	2.629
95	.845	1.291	1.661	1.985	2.366	2.629
96	.845	1.290	1.661	1.985	2.366	2.628
97	.845	1.290	1.661	1.985	2.365	2.627
98	.845	1.290	1.661	1.984	2.365	2.627
99	.845	1.290	1.660	1.984	2.364	2.626
100	.845	1.290	1.660	1.984	2.364	2.626
∞	.842	1.282	1.645	1.960	2.326	2.576